15. Sh. U. Galiev, Nonlinear Waves in Finite Continua [in Russian], Naukova Dumka, Kiev (1988).
16. V. V. Selivanov, V. S. Solov'ev, and N. N. Sysoev, Shock and Detonation Waves. Methods of Investigation [in Russian], Izd. MGU, Moscow (1990).
17. A. B. Kiselev and M. V. Yumashev, "Numerical study of the shock compression of a micropore in a thermoelastoviscoplastic material," Vestn. Mosk. Univ. Mat. Mekh., No. 1 (1992).
18. M. L. Wilkins, "Modelling the behavior of materials," Struct. Impact and Crashworthiness. Proc. Int. Conf., Vol. 2, London; New York (1984).
19. A. B. Kiselev and M. V. Yumashev, "Criteria of the dynamic failure of a thermoelastoplastic medium," Vestn. Mosk. Univ. Mat. Mekh., No. 4 (1990).
20. B. E. Pobedrya, "Fracture criteria of structurally nonuniform materials," in: Plastic Deformation and Fracture of Solids [in Russian], Nauka, Moscow (1988).
21. A. I. Gulidov and V. M. Fomin, "Numerical modelling of the rebound of axisymmetric rods from a solid barrier," Prikl. Mekh. Tekh. Fiz., No. 3 (1980).
22. A. B. Kiselev, "Numerical study, in a three-dimensional formulation, of the collision of elastoplastic bodies with a rigid barrier," Vestn. Mosk. Univ. Mat. Mekh., No. 4 (1985).
23. P. Pezhina, Basic Principles of Viscoplasticity, [Russian translation], Mir, Moscow (1968).
24. V. M. Boiko, A. I. Gulidov, A. I. Papyrin, et al., "Experimental-theoretical study of the rebound of short rods from a solid barrier," Prikl. Mekh. Tekh. Fiz., No. 5, (1982).
25. S. P. Kiselev, V. M. Fomin, and Yu. A. Shitov, "Numerical modelling of the rebound of a porous cylinder from a rigid barrier," Prikl. Mekh. Tekh. Fiz., No. 3 (1990).

LAWS OF THE KINETIC THEORY OF STRENGTH OF FROZEN SOILS

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UDC 624.139

The studies [1, 2], devoted to the rheology of frozen soils, demonstrated the kinetic nature of the strength of these soils and presented a physical interpretation of the parameters of an equation describing their long-term strength from an atomic-kinetic viewpoint. However, these findings have not found expression in the mathematical theory of the rheology of frozen soils. Nor has use been made of quantitative relations and parameters expressing the temperature-time dependence of strength. According to [3] such relations and parameters can be established for all solids. Standard interpolation formulas which include empirical coefficients serve as the basis of quantitative methods of determining the strength characteristics of frozen soil. In particular, use is made of the coefficient [1]:

$$\tau = (g/\sigma)^{1/\Gamma}. \tag{1}$$

Here, τ is the time to failure (life); σ is pressure; g and Γ are empirical coefficients dependent on temperature, soil composition, type of load, etc.

We will attempt to find the temperature dependence of the strength of frozen soil in explicit form. Equation (1) is conveniently represented as

$$\tau = \tau_0(\sigma_m/\sigma)^{1/\Gamma}, \tag{2}$$

where σ_m is the instantaneous (maximum) strength corresponding to the minimum life ("moment") τ_0. In accordance with the representations of atomic-kinetic theory, this physical "moment" is equal to the period of thermal vibration of the atoms $\tau_0 \approx 10^{-13}$ sec.

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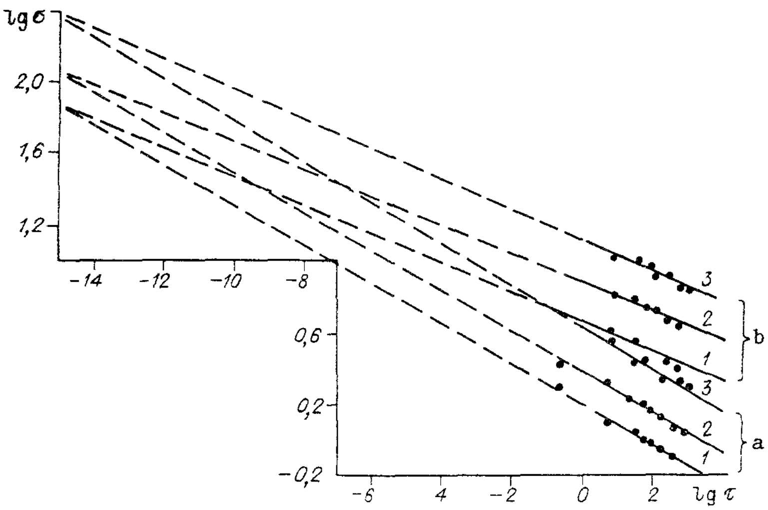

Fig. 1

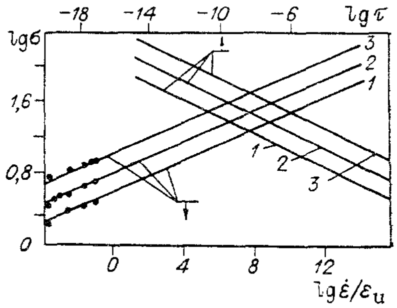

Fig. 2

The following was found from an analysis of actual data from tests of frozen soils in which the variables were soil composition, temperature, stress state, the material and form of the foundation, and other quantities [4].

First of all, the graphs of the dependence of log σ on log τ for frozen soils tested at one temperature while the other conditions were varied converged to a single point — a pole with the coordinates $\tau_0 \approx 10^{-13}$ sec ($10^{-14.8}$ min) and $\sigma = \sigma_m$. Secondly, the time dependences of the strength of frozen soils tested under the same conditions except for temperature do not converge to a single point but are instead parallel. This is apparent from Fig. 1, which shows experimental data on the strength of frozen soil in compression (a) and frozen clay in adhesion tests (b) at different temperatures. This data, borrowed from [1], was analyzed using Eq. (2) (lines 1-3 are for t = −5, −10, −20°C). Similar results were obtained when we used Eq. (2) to analyze test data from [4]. This indicates that the slope is independent of temperature but is a function of all other conditions. Temperature affects the second parameter of the strength equation g and the second coordinate of the pole σ_m entering into the expression for this parameter.

Thus, quantitative manifestations of the kinetic nature of the strength of frozen soils have certain specific features connected with the fact that frozen soil, rather than being a solid, is a complex system consisting of an organic-mineral skeleton, water in three phases, and aqueous solutions and gases. Also — and this is a key point — the temperature of frozen soil is very close to the melting point.

The main type of bond between particles in frozen soil is icy-cementational. Thus, it can be assumed that an elementary volume of frozen soil, i.e., a volume whose dimensions can be ignored, instantaneously loses its strength at the melting point of ice. Here, instantaneous strength is equal to the pressure at which the ice melts. The melting point of ice depends on external pressure in accordance with the Clapeyron—Clausius law. Then

$$\sigma_m = 0.1 + t/K \approx t/K \tag{3}$$

(t is temperature, °C; K ≈ 0.073°C/MPa).

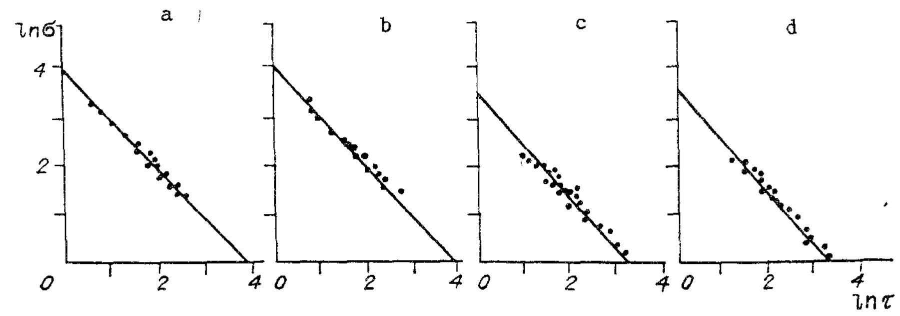

Fig. 3

As is known, when soil freezes (or thaws) as moisture migrates in an open system, the relationship between external pressure and the temperature of the phase transitions which take place in the soil moisture is described by the so-called generalized Clapeyron–Clausius law. In the expression of this law, K is roughly ten times greater than in the above case.

According to experimental data, the average value of K is about 0.079°C/MPa [4]. This is close to the value of the coefficient in the classical Clapeyron–Clausius equation. Thus, in the example cited (see Fig. 1), $K = t/\sigma_m$ fluctuates within the range 0.073-0.083°C/,Pa for different conditions. Allowing for the migration of moisture in the determination of σ_m undoubtedly refines this value. However, this factor is not always manifest, given the logarithmic accuracy of Eq. (2).

Of course, all of the above considerations are based on a very long-range time extrapolation of empirical data - although similar doubts attend the generally accepted extrapolation of (for example) the results of 8 h tests of frozen soil to a 50 yr period. It can also probably be argued that the results being discussed here are also insufficiently representative. Nevertheless, it is evident that, with a decrease in τ, the function σ(τ) approaches a common pole with the coordinates $\tau_0 \approx 10^{-13}$ sec and $\sigma_m = t/K$ (in the logarithmic scale). It is also necessary to take into account the strict physical meanings of τ_0 and σ_m, the convenience for experimental work and calculations (experimentally, it is necessary to find only one parameter – Γ – which is independent of temperature) and the relatively small effect of τ_0 on σ (at Γ = 0.1, a decrease in τ_0 by a factor of 1000 results in only a doubling of maximum long-term strength). All this makes it possible to recommend Eqs. (2) and (3) for practical use.

It should be noted that the above-described laws turn out to be valid in most cases where actual materials are concerned. In particular, in tests with a spherical die, the pole of the dependence of σ on τ often does not coincide with the coordinates $\tau_0 = 10^{-13}$ sec and $\sigma_m = t/K$. In an analysis of the results of tests involving the driving of model piles into frozen soil with a temperature of –2.2°C in [5], it was established that the graphs of the dependence of log σ on log τ consist of two linear section intersecting at σ = 0.1 MPa. It is interesting that the dependence of σ on τ also satisfies (2)-(3) at σ > 0.1. In fact, the parameters of interpolation formula (1) in [5] have the form g = 0.33, Γ = 0.145. Having inserted the experimental data into the formula for $g = \tau_0^{\Gamma} t/K$, we obtain $g = 10^{-13 \cdot 0.145} \times 2.2/0.079 = 0.36$. The values of g calculated from the formula which follows from (2) and the experimental values of g differ by only 9%.

It was shown in [6] for frozen soils that if the load, instead of being applied to the entire specimen, is applied only to the part occupied by particles of soil and ice – as opposed to the part occupied by unfrozen water – (we will designate this load as $\bar{\sigma}$), then the pole of the curves of the dependence of log $\bar{\sigma}$ on log τ is independent of t. This is consistent with out finding on the dependence of σ_m on t.

The fact is that $\bar{\sigma}_m = \sigma_m/K_n$ (K_n is a coefficient which is less than 1 and is directly proportional to temperature [6]). At the same time, σ_m is also directly proportional to t. As a result, it turns out that $\bar{\sigma}_m$ is nearly independent of temperature, since it goes into the numerator and the denominator of the expression of this quantity.

Determining the time dependence of strength for specific materials is not sufficient for systematically analyzing deviations of this dependence from Eq. (2). This problem requires further studies.

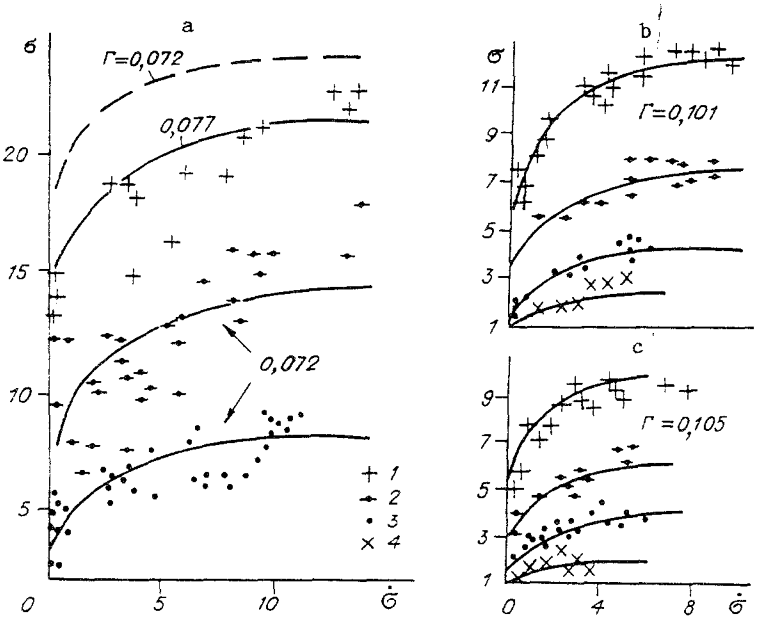

Fig. 4

In accordance with the kinetic concept of the strength of solids, the limiting strain ε_u (at which the solid suffers a loss of integrity and fails) is equal to the product of creep rate $\dot{\varepsilon}$ and lifetime and is a constant:

$$\varepsilon_u = \dot{\varepsilon}\tau = \text{const.} \tag{4}$$

Since the fracture energy of a solid is approximately equal to the energy of sublimation (at which the body vaporizes and disappears) then $\varepsilon_u = 1$ [3]. It follows from (4) that the dependence of creep rate on the load is a "mirror" reflection of the dependence of lifetime on the load.

It was established experimentally in [1] that Eq. (4) is also generally valid for frozen soils. The limiting strain of a frozen soil depends very slightly on temperature and load and somewhat more on the type of stress state and type of soil. Meanwhile, the mean value of this strain is close to the relative subsidence accompanying the transformation of ice into water ($\varepsilon_u \approx 0.09$).

We find the expression for the relative creep rate from a combination of the solutions of (2) and (4):

$$\frac{\dot{\varepsilon}}{\varepsilon_u} = \frac{1}{\tau} = \frac{v}{s} = \tau_0^{-1}(\sigma/\sigma_m)^{1/\Gamma} \tag{5}$$

(s and v are dimensional subsidence and creep rate).

The validity of the solution [5] – being (as in the case of solids) a "mirror" reflection of the dependence of lifetime on load – has been shown by experimental results. As an example, Fig. 2 shows graphs of the dependence of the logarithms of lifetime and creep rate on the logarithms of the load on frozen sandy loam with a moisture content of 26%. The values shown are the actual values from [1] and values calculated from Eqs. (2) and (5) with $g = 0.16$ and $\Gamma = 0.1$ (lines 1-3 are for $t = -5, -10, -20$°C, respectively). The reciprocity of Eqs. (2) and (5) is also apparent from the graphs presented in [5]. We can use (5) to find an expression which connects strength with dimensionless and dimensional steady-state creep rate

$$\sigma = (\tau_0\dot{\varepsilon}/\varepsilon)^{\Gamma}\, t/K,\ \sigma = (\tau_0 v/s)^{\Gamma}\, t/K. \tag{6}$$

In complex cases in which there is some doubt as to the validity of using the kinetic concept of strength, it is best to express (6) in a general form analogous to (1):

$$\sigma = g_\varepsilon \dot{\varepsilon}^\Gamma, \ \sigma = g_v v^\Gamma \tag{7}$$

(g_ε, g_V are parameters determined from testing). Equations (6) and (7) are convenient because they make it possible to perform calculations based both on strength and on strain.

The quantitative features observed in tests with a constant load are also seen in tests with a variable load – in particular, a load which changes with a constant rate $\dot{\sigma}$.

We find analytical equations for the rate dependence of strength and lifetime from the solution of a system comprised of the equation describing lifetime under a constant load (2), an equation expressing the well-known [1, 3] principle of the additivity of perturbations

$$\int_{\tau_0}^{\tau} \{\tau [\sigma(\tau)]\}^{-1} d\tau = 1 \tag{8}$$

and the equation for a load changing at a constant rate:

$$\sigma = \dot{\sigma}\tau. \tag{9}$$

After some simple transformations, we obtain

$$\tau = (g/\dot{\sigma})^\alpha \alpha^{-\alpha}; \tag{10}$$

$$\sigma = g^{\alpha/\Gamma} \dot{\sigma}^\alpha \alpha^{-\alpha}, \tag{11}$$

where, as before, $g = \tau_0{}^\Gamma \sigma_m$; $\alpha = \Gamma/(1 + \Gamma)$.

Equations (10) and (11) are valid for the case when the total load leading to fracture is greater than the limiting long-term load. The use of these formulas can save a significant amount of time during testing.

In tests we conducted with S. M. Pakhomov, we subjected to groups of specimens of loamy clay to uniaxial compression. Moisture content and temperature were 20% and −3.3°C for the first group and 24% and −3.9°C for the second group. We also tested two groups of specimens of ice in the form of cubes 10 cm high. These cubes were reinforced with an interlayer of dornite to depths of 7 and 3 cm (groups 3 and 4). The temperature of these specimens was −3.4°C. The test results were analyzed in the form of the dependence of ln τ on ln $\dot{\sigma}$ (Fig. 3, where a-d correspond to groups 1-4). We then used the above-described method to determine g and Γ:

$$g_1 \ldots g_4 = 4.2;\ 5.7;\ 2.8;\ 2.7 \ \mathrm{MPa/min^{-\Gamma}},$$
$$\Gamma_1 \ldots \Gamma_4 = 0.07;\ 0.07;\ 0.074;\ 0.076.$$

There are five quantities in the expression for g. All of them are known either as constants (τ_0 and K) or empirical results. If any four of them are taken from experimental data and the fifth – temperature, for example – is calculated and compared with the test temperature, it becomes obvious that the test results are consistent with the kinetic concept of strength when the two indicated values of temperature are the same.

We used Eqs. (7) and (11) to find the temperature in four tests: $t_1 \ldots t_4$ = −4.1; −4.6; −2.7; −2.8°C.

Considering that τ_0 and K were evaluated on the basis of theoretical considerations, we can regard the resulting difference (18-25%) between the calculated and experimental values of temperature (−3.9 and −3.4°C) as wholly satisfactory.

Figure 4 (a-c: sand, sandy loam, and loamy clay, respectively) shows the results of tests [7] of different types of soils in uniaxial compression at different temperatures and loading rates. The results are depicted in the form of the rate dependence of strength. The lines in the graphs were constructed from results calculated on the basis of Eq. (11). Here, we first used the method described above to find the values of g and Γ (points 1-4 correspond to t = −15, −8, −4, −2°C, respectively). The good agreement between the actual and calculated data is evident, which illustrates the agreement between the rate dependence of strength and the kinetic concept of strength. As in the case of a constant load, it can be assumed that Γ is independent of t — regardless of temperature, each soil corresponds to a single value of Γ (except for sand with $t = -15°C$, this result apparently being connected with a deviation from the pressure dependence of the freezing point of water described by Eq. (3)).

Although the form of the temperature-time (or rate) dependence of strength will be refined later, it can be taken as an established fact that temperature is connected with only one of the two parameters of the rheological curve. This makes it possible to significantly shorten and simplify tests and calculations performed to determine the strength of frozen soil.

LITERATURE CITED

1. S. S. Vyalov, Rheological Principles of Soil Mechanics [in Russian], Vyssh. Shk., Moscow (1978).
2. Yu. K. Zaretskii and B. D. Chumichev, Short-Term Creep of Ice [in Russian], Nauka, Novosibirsk (1982).
3. V. R. Regel', A. I. Slutsker, and E. V. Tomashevskii, Kinetic Nature of the Strength of Solids, [in Russian], Nauka, Moscow (1974).
4. A. A. Konovalov, "Quantitative approximation of the temperature and time dependence of the strength of frozen soils," Izh. Geologiya, No. 2 (1986).
5. A. S. Gerasimov, "Evaluating the long-term strength of frozen soil," in: Problems of Soil Mechanics and Engineering Geocryology [in Russian], Stroizdat, Moscow (1990).
6. L. T. Roman, "Predicting the long-term strength of frozen peat soils," ibid.
7. A. G. Deryugin, "Study of the rate dependence of frozen soils," in: Engineering Studies of Frozen Soils [in Russian], Nauka, Novosibirsk (1979).

PROBLEM OF THE THEORY OF BEAMS WITH INITIAL STRESSES

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UDC 539.3

Aysymptotic averaging methods — which have been widely used for monolithic composites (see [1-3] and the accompanying bibiliographies) — are now being used to study bodies with a periodic structure that occupy thin regions: plates and beams [4-6]. In the present study, we make a transition from a three-dimensional problem of the theory of elasticity with initial stresses in the region of the small diameter ε (which is formalized in the form $\varepsilon \to 0$) to a problem of beam theory. In the general case, the new problem (which is asymptotically exact) differs from the classical problem. It coincides with the classical problem for uniform beams, however, i.e., the difference between the asymptotic and classical theories is seen for beams of complex structure. The use of such beams in modern structures makes the corrections introduced by asympototic theory practically important. The difference between the given problem and the problem examined in [6] is the asymmetry of the coefficients. This leads to the appearance of new elements in the use of asymptotic methods, as well as to several new cellular problems. As will be seen from the below discussion ε, the order of the initial stresses σ^*_{ij} relative to the diameter of the region ε plays a significant role in the problem. To account for this, we take the initial stresses in the form $\sigma^*_{ij} = \varepsilon^{-2}\sigma^{*(-2)}_{ij} + \varepsilon^{-1}\sigma^{*(-1)}_{ij} + \ldots$, corresponding to bending of the beam or its axial tension with fixed forces. The axial tension of a beam with fixed strains, when σ^*_{ij} is on the order of ε^{-4}, leads to results similar to [7-9] for monolithic bodies. This case is not examined here.

Formulation of the Problem. We will examine a body of periodic construction obtained by repeating a certain unit cell (UC) P_ε (ε is its characteristic dimension) along the Ox_1 axis (see Fig. 1). As a result, we have a body of periodic structure with the character-

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